Soil Temperature and Heat Flow

A. Introduction

Soil temperature, its value at any moment and the manner with which it varies in time and space, is a factor of primary importance in determining the rates and directions of soil physical processes and of energy and mass exchange with the atmosphere—including evaporation and aeration. Temperature also governs the types and rates of chemical reactions which take place in the soil. Finally, soil temperature strongly influences biological processes, such as seed germination, seedling emergence and growth, root development, and microbial activity.

Soil temperature varies in response to changes in the radiant, thermal, and latent energy exchange processes which take place primarily through the soil surface. The effects of these phenomena are propagated into the soil profile by a complex array of transport processes, the rates of which are affected by time-variable and space-variable soil properties. Hence the quantitative formulation and prediction of the soil thermal regime can be a formidable task. Even beyond passive prediction, the possibility of actively controlling or modifying the thermal regime requires thorough knowledge of the processes at play and of the environmental and soil parameters which govern their rates. The pertinent soil parameters include the specific heat capacity, thermal conductivity, and thermal diffusivity (all of which are strongly affected by bulk density and wetness), as well as the internal sources and sinks of heat operating at any time.

Present day theory can provide at least a semiquantitative interpretation...
of observed influences of soil surface conditions, including the presence of mulching materials and various tillage treatments on the soil's thermal regime. Moreover, available theory can help to explain why the annual temperature variation penetrates into the soil much more deeply than the diurnal variation; it can account for the obvious difference in temperature distribution among soils of differing constitution, such as sand, clay, or peat. It can also explain why the surface of a dry soil exhibits high maxima and low minima temperatures, and suggest how these extremes may be moderated when the soil moisture content is changed. Theories are now being developed to deal with freezing and thawing phenomena in cold-region soils. Finally, the important interaction of heat flow and water flow is being examined in an effort to understand how the transports of matter and energy occur simultaneously and independently in the soil.

Reviews of soil temperature and heat flow have been published over the years by Kerssen (1949), Hasan (1952), van Rooyen and Winterkorn (1959), Smith et al. (1964), Taylor and Jackson (1965), van Wijk (1963), Chudnovskii (1966), van Ravel (1972), and de Vries (1975).

B. Modes of Energy Transfer

We begin with some basic physics. In general, there are three principal modes of energy transfer: radiation, convection, and conduction.

By radiation, we refer to the emission of energy in the form of electromagnetic waves from all bodies above 0 K. According to the Stefan-Boltzmann law, the total energy emitted by a body \( J \) integrated over all wavelengths, is proportional to the fourth power of the absolute temperature \( T \) of the body's surface. This law is usually formulated as

\[
J = \sigma T^4
\]  

(9.1)

where \( \sigma \) is a constant and \( \varepsilon \) is the emissivity coefficient which equals unity for a perfect emitter (generally called a black body). The absolute temperature also determines the wavelength distribution of the emitted energy. Wien's law states that the wavelength of maximal radiation intensity \( \lambda_m \) is inversely proportional to the absolute temperature:

\[
\lambda_m = \frac{2900}{T}
\]  

(9.2)

where \( \lambda_m \) is in microns. The actual intensity distribution as a function of wavelength and temperature is given by Planck's law:

\[
E_\lambda = \frac{C_1}{\lambda^5} \left[ \exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]
\]  

(9.3)

where \( E_\lambda \) is the energy flux emitted in a particular wavelength range, and
$C_1$, $C_2$ are constants. Since the temperature of the soil surface is generally of the order of 300 K (though it can range, of course, from below 273 K, the freezing point, to 330 K or even higher), the radiation emitted by the soil surface has its peak intensity at a wavelength of about 10 $\mu$m and its wavelength distribution over the range of 3–50 $\mu$m. This is in the realm of infrared, or heat, radiation. A very different spectrum is emitted by the sun, which acts as a black body at an effective surface temperature of about 6000 K. The sun’s radiation includes the visible light range of 0.3–0.7 $\mu$m, as well as some infrared radiation of greater wavelength (up to about 3 $\mu$m) and some ultraviolet radiation ($\lambda < 0.3 \mu m$). Since there is very little overlap between the two spectra, it is customary to distinguish between them by calling the incoming solar spectrum short-wave radiation, and the spectrum emitted by the earth long-wave radiation.

The second mode of energy transfer, called convection, involves the movement of a heat-carrying mass, as in the case of ocean currents or atmospheric winds. An example more pertinent to soil physics would be the infiltration of hot waste water (from, say, a power plant) into an initially cold soil.

Conduction, the third mode of energy transfer, is the propagation of heat within a body by internal molecular motion. Since temperature is an expression of the kinetic energy of a body’s molecules, the existence of a temperature difference within a body will normally cause the transfer of kinetic energy by the numerous collisions of rapidly moving molecules from the warmer region of the body with their neighbors in the colder region. The process of heat conduction is thus analogous to the process of diffusion, and in the same way that diffusion tends in time to equilibrate a mixture’s composition throughout, heat conduction tends to equilibrate a body’s internal distribution of molecular kinetic energy—that is to say, its temperature.

In addition to the three modes of energy transfer described, there is a composite phenomenon which one may recognize as a fourth mode, namely the latent heat transfer. A prime example is the process of distillation, which includes the heat-absorbing stage of evaporation, followed by the convective or diffusive movement of the vapor, and ending with the heat-releasing stage of condensation. A similar catenary process can also occur in transition back and forth from ice to liquid water.

The transfer of heat through the soil surface may occur by any or all of the above mechanisms. Within the soil, however, heat transfer by radiation, convection, and distillation are generally of secondary importance, and the primary process of heat transport is by molecular conduction.
C. Conduction of Heat in Soil

The conduction of heat in solids was analyzed as long ago as 1822 by Fourier, whose name is associated with the linear transport equations which have been used ever since to describe heat conduction. These equations are mathematically analogous to the diffusion equations (Fick's laws) as well as to Darcy's law for the conduction of fluids in porous media. An analogy can also be drawn between these laws and Ohm's law for the conduction of electricity (See Chapter 6). A definitive text on the mathematics of heat conduction was published by Carslaw and Jaeger (1959).

The first law of heat conduction, known as Fourier's law, states that the flux of heat in a homogeneous body is in the direction of and proportional to the temperature gradient:

\[ q_x = - \kappa \nabla T \quad (9.4) \]

where \( q_x \) is the thermal flux (i.e., the amount of heat conducted across a unit cross-sectional area in unit time), \( \kappa \) is thermal conductivity, and \( \nabla T \) the spatial gradient of temperature. In one-dimensional form, this law is written

\[ q_x = - \kappa \frac{dT}{dx} \quad \text{or} \quad q_x = - \kappa \frac{dT}{dz} \quad (9.5) \]

Here \( \frac{dT}{dx} \) is the temperature gradient in any arbitrary direction designated \( x \), and \( \frac{dT}{dz} \) is, specifically, the vertical direction representing soil depth (\( z = 0 \) being the soil surface). The subscripts attached to the thermal conductivity term are meant to account for the possibility that this parameter may have different values in different directions. The negative sign in these equations is due to the fact that heat flows from a higher to a lower temperature (i.e., in the direction of a negative temperature gradient).

If \( q_x \) is expressed in calories per square centimeter per second and the temperature gradient in degrees Kelvin per centimeter, \( \kappa \) has the units of calories per centimeter-degree-second. If, on the other hand, the thermal flux is given in watts per meter and the gradient in degrees per meter, the thermal conductivity assumes the units of watts per meter-degree.

Equation (9.4) is sufficient to describe heat conduction under steady-state conditions. But it is to say where the temperature at each point in the conducting medium and the flux remain constant in time. To account for nonsteady or transient conditions, we need a second law analogous to Fick's second law of diffusion as embodied in Eq. (7.15). To obtain the second law of heat conduction, we invoke the principle of energy conservation in the form of the continuity equation, which states that, in the absence of any sources or sinks of heat, the time rate of change in heat content of a volume element of the conducting medium (in our case, soil) must equal the change of flux with distance.
\[ \rho c_p \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q}_h, \]  
\[ \text{Eq. (9.6)} \]

where \( \rho \) is mass density and \( c_p \), specific heat capacity per unit mass, (called simply specific heat and defined as the change in heat content of a unit mass of the body per unit change in temperature). The product \( \rho c_p \) (often designated \( C \)) is the specific heat capacity per unit volume, and \( \frac{\partial T}{\partial t} \) is the time rate of temperature change. Note that the symbol \( \rho \) represents the total mass per unit volume, including the mass of water in the case of a moist soil. The symbol \( V \) (curl) is the shorthand representation of the three-dimensional gradient. An equivalent form of Eq. (9.6) is

\[ \rho c_p \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q}_h + \nabla (\rho c_p \nabla T) \]

where \( \nabla \cdot \mathbf{q}_h \) is the orthogonal direction coordinate.

Combining Eqs. (9.4) and (9.6), we obtain the desired second law of heat conduction:

\[ \rho c_p \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q}_h (x \nabla T) \]  
\[ \text{Eq. (9.7)} \]

which in one-dimensional form is

\[ \rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \rho c_p \frac{\partial T}{\partial x} \right) \]  
\[ \text{Eq. (9.8)} \]

Sometimes we may need to account for the possible occurrence of heat sources or sinks in the realm where heat flow takes place. Heat sources include such phenomena as organic matter decomposition, settling of initially dry soil material, and condensation of water vapor. Heat sinks are generally associated with evaporation. Lumping all these sources and sinks into a single term \( S \), we can rewrite the last equation in the form

\[ \rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \rho c_p \frac{\partial T}{\partial x} \right) + S(x,t) \]  
\[ \text{Eq. (9.9)} \]

in which the source–sink term is shown as a function of both space and time.

The ratio of the thermal conductivity \( k \) to the volumetric heat capacity \( C = \rho c_p \) is called the thermal diffusivity, designated \( D_\| \). Thus,

\[ D_\| = \frac{k}{C} \]  
\[ \text{Eq. (9.10)} \]

Substituting \( D_\| \) for \( k \), we can rewrite Eqs. (9.5) and (9.8):

\[ q_h = -D_\| \frac{\partial T}{\partial x} \]  
\[ \text{Eq. (9.11)} \]

and

\[ \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( D_\| \frac{\partial T}{\partial x} \right) \]  
\[ \text{Eq. (9.12)} \]
V. COMPOSITE PROPERTIES AND BEHAVIOR

In the special case where \( D_t \) can be considered constant, i.e., not a function of distance \( x \), we can write

\[
\frac{dT}{\partial t} = D_t \frac{\partial^2 T}{\partial x^2}
\]

(9.13)

To solve the foregoing equation so as to obtain a description of how temperature varies in both space and time, we need to know, by means of measurement or calculation, the pertinent values of the three parameters just defined, namely, the volumetric heat capacity \( C_v \), thermal conductivity \( k \), and thermal diffusivity \( D_t \). Together, they are called the thermal properties of soils.

D. Volumetric Heat Capacity of Soils

The volumetric heat capacity \( C_v \) of a soil is defined as the change in heat content of a unit bulk volume of soil per unit change in temperature. Its units are calories per cubic centimeter per degree (Kelvin), or joules per cubic meter per degree. As such, \( C_v \) depends on the composition of the soil's solid phase (the mineral and organic constituents present), bulk density, and the soil's wetness (see Table 9.1).

The value of \( C_v \) can be calculated by addition of the heat capacities of the various constituents, weighted according to their volume fractions. As given by de Vries (1973),

\[
C_v = \sum f_j C_{v_j} + f_s C_s + f_a C_a
\]

(9.14)

Here, \( f_j \) denotes the volume fraction of each phase: solid (subscripted \( s \)), water (\( w \)), and air (\( a \)). The solid phase includes a number of components, subscripted \( j \), such as various minerals and organic matter; and the symbol \( \sum \) indicates the summation of the products of their respective volume fractions and heat capacities. The \( C \) value for water, air, and each component of the solid phase is the product of the particular density and specific heat per unit mass (i.e., \( C_w = \rho_w C_{w,m} \); \( C_s = \rho_s C_{s,m} \); \( C_a = \rho_a C_{a,m} \)).

Most of the minerals composing soils have nearly the same values of density (about 2.65 g/cm\(^3\) or 2.65 x 10\(^3\) kg/m\(^3\)) and of heat capacity (0.48 cal/cm\(^3\)-K or 2.3 x 10\(^3\) J/m\(^3\)-K). Since it is difficult to separate the different kinds of organic matter present in soils, it is tempting to lump them all into a single constituent (with an average density of about 1.3 g/cm\(^3\) or 1.3 x 10\(^3\) kg/m\(^3\); and an average heat capacity of about 0.6 cal/cm\(^3\)-K or 2.5 x 10\(^3\) J/m\(^3\)-K). The density of water is less than half that of mineral matter (1 g/cm\(^3\) or 1.0 x 10\(^3\) kg/m\(^3\)) but the specific heat of water is more than twice as large (1 cal/cm\(^3\)-K or 4.2 x 10\(^3\) J/m\(^3\)-K). Finally, since the density of air is only about 1.100 that of water, its contribution to the specific heat of the composite soil can generally be neglected.
Thus, Eq. (9.14) can be simplified as follows:

\[ C = f_a C_a + f_m C_m + f_w C_w \]  

(9.15)

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Density ( \rho ) (gm cm(^{-3}))</th>
<th>Heat capacity ( C ) (cal cm(^{-3}) K(^{-1}))</th>
<th>( \frac{C}{(W m^{-2} K)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>2.60</td>
<td>2.86 \times 10^5</td>
<td>4.88</td>
</tr>
<tr>
<td>Other minerals (average)</td>
<td>2.65</td>
<td>2.65 \times 10^5</td>
<td>0.48</td>
</tr>
<tr>
<td>Organic matter</td>
<td>1.3</td>
<td>1.3 \times 10^5</td>
<td>0.6</td>
</tr>
<tr>
<td>Water (liquid)</td>
<td>1.0</td>
<td>1.0 \times 10^3</td>
<td>1.0</td>
</tr>
<tr>
<td>Ice</td>
<td>0.92</td>
<td>0.92 \times 10^3</td>
<td>0.45</td>
</tr>
<tr>
<td>Air</td>
<td>0.00125</td>
<td>1.25</td>
<td>0.003</td>
</tr>
</tbody>
</table>

where subscripts m, o, w refer to mineral matter, organic matter, and water, respectively. Note that \( f_m + f_o + f_w = 1 - f_a \), and the total porosity \( f = f_l + f_a \). The reader will recall that in our preceding chapters we designated the volume fraction of water \( f_w = 0 \). Knowing the approximate average values of \( C_m, C_o, \) and \( C_w \) \( (0.46, 0.60, \) and \( 1.0 \) cal gm, respectively), we can further simplify Eq. (9.15) to give

\[ C = 0.48 f_m + 0.60 f_o + f_w \]  

(9.16)

The use of Eq. (9.15) must be qualified in the case of frozen or partially frozen soils, since the properties of ice differ somewhat from those of liquid water \( (\rho = 0.92 \) gm cm\(^{-3}\), \( 0.92 \times 10^5 \) kg m\(^{-3}\), and \( C = 0.45 \) cal cm\(^{-3}\) K or \( 1.9 \times 10^5 \) J m\(^{-3}\) K\(^{-1}\) ).

In typical mineral soils, the volume fraction of solids is in the range of 0.45–0.65, and \( C \) values range from less than 0.25 cal cm\(^{-3}\) K (about 1 MJ m\(^{-3}\) K) in the dry state to about 0.75 cal cm\(^{-3}\) K (\( \times 3 \) MJ m\(^{-3}\) K) in the water-saturated state.

Apart from the method described for calculating a soil's volumetric heat capacity, it is, of course, also possible to measure it using calorimetric techniques (Taylor and Jackson, 1965).

E. Thermal Conductivity of Soils

Thermal conductivity, designated \( k \), is defined as the amount of heat transferred through a unit area in unit time under a unit temperature gradient. As shown in Table 9.2, the thermal conductivities of specific soil
Table 9.2
THERMAL CONDUCTIVITIES OF SOIL CONSTITUENTS (at 10 °C) AND AT 32°C (at 0°C)

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Soil (°C)</th>
<th>Water (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>21</td>
<td>0.6</td>
</tr>
<tr>
<td>Other minerals (average)</td>
<td>7</td>
<td>0.2</td>
</tr>
<tr>
<td>Organic matter</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Water (liquid)</td>
<td>1.36</td>
<td>0.59</td>
</tr>
<tr>
<td>Ice</td>
<td>3.2</td>
<td>2.7</td>
</tr>
<tr>
<td>Air</td>
<td>0.06</td>
<td>0.625</td>
</tr>
</tbody>
</table>

constituents differ very markedly (see also Table 9.3). Hence the space-average (macroscopic) thermal conductivity of a soil depends upon its mineral composition and organic matter content, as well as on the volume fractions of water and air. Since the thermal conductivity of air is very much smaller than that of water or solid matter, a high air content (or low water content) corresponds to a low thermal conductivity. Moreover, since the proportions of water and air vary continuously, x is also time variable. Soil composition is seldom uniform in depth, hence x is generally a function of depth as well as of time. It also varies with temperature, but under normal

Table 9.3
AVERAGE THERMAL PROPERTIES OF SOILS AND SNOW

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Porosity (%)</th>
<th>Volume density (g/cm³)</th>
<th>Thermal conductivity (W/m-°C)</th>
<th>Volumetric heat capacity (J/g-°C)</th>
<th>Damping depth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>0.4</td>
<td>1.6</td>
<td>0.9</td>
<td>0.3</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.2</td>
<td>4.7</td>
<td>0.5</td>
<td>15.2</td>
</tr>
<tr>
<td>Clay</td>
<td>0.6</td>
<td>0.6</td>
<td>4.6</td>
<td>0.3</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.2</td>
<td>28</td>
<td>0.5</td>
<td>12.4</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.4</td>
<td>3.8</td>
<td>0.7</td>
<td>15.2</td>
</tr>
<tr>
<td>Peat</td>
<td>0.8</td>
<td>0.8</td>
<td>0.14</td>
<td>0.3</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.4</td>
<td>6.7</td>
<td>0.35</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>0.6</td>
<td>1.2</td>
<td>1.15</td>
<td>5.4</td>
</tr>
<tr>
<td>Snow</td>
<td>0.95</td>
<td>0.95</td>
<td>0.15</td>
<td>0.06</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>0.2</td>
<td>0.32</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.3</td>
<td>1.2</td>
<td>0.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>

* After van Weeck and van Vries (1955).
conditions this variation is ignored. The factors which affect thermal conductivity \( \kappa \) are the same as those which affect the volumetric heat capacity \( C \), but the measure of their effect is different so that the variation in \( \kappa \) is much greater than that of \( C \). In the normal range of soil moisture experienced in the field, \( C \) may undergo a threefold or fourfold change, whereas the corresponding change in \( \kappa \) may be hundredsfold or more. One complicating factor is that, unlike heat capacity, thermal conductivity is sensitive not merely to the volume composition of a soil but also to the sizes, shapes, and spatial arrangements of the soil particles.

The problem of expressing the overall thermal conductivity of a soil as a function of the specific conductivities and volume fractions of the soil's constituents is very intricate, as it involves the internal geometry of soil structure and the transmission of heat from particle to particle and from phase to phase.

Two relatively simple alternative cases can be envisaged: a dry soil or a water-saturated soil with the same particle configuration. In either case, we have a two-phase system in which the particles are dispersed in a continuous medium of fluid (air or water) with a volume fraction \( f_p \) and thermal conductivity \( \kappa_p \). The particles then occupy a volume fraction \( f_s = 1 - f_p \) and have a thermal conductivity \( \kappa_s \). A composite thermal conductivity for the medium as a whole can be defined as follows: Consider a representative cube of soil with side \( l \) large in comparison with the diameters of the particles and pores. Assume that the upper face is at a temperature \( T_1 \) and the bottom face at a lower temperature \( T_2 \). A constant heat flux \( q_h \) will then pass through the cube proportional to the overall temperature gradient, with \( \kappa_c \) as the factor of proportionality for the composite medium:

\[
q_h = -\kappa_c \frac{dT}{dx} = \kappa_c (T_1 - T_2) / l
\]

Since the cube is a mixture of two phases, the composite thermal conductivity \( \kappa_c \) will be intermediate between \( \kappa_p \) and \( \kappa_s \). According to Burger (1915),

\[
\kappa_c = (f_p \kappa_p + f_s \kappa_s) / (f_p \kappa_p + f_s \kappa_s)
\]

wherein the factor \( k \) is the ratio of the average temperature gradient in the particles to the corresponding gradient in the continuous fluid:

\[
k = \frac{(dT / dx)_p}{(dT / dx)_c}
\]

According to de Vries (1975) the value of \( k \) depends not only on the ratio \( \kappa_s / \kappa_p \), but also on the particle sizes, shapes, and mode of packing: These variables are difficult to characterize quantitatively in the case of particles of irregular shape, distribution of sizes, and packing arrangements.
If there are several types of particles with different shapes or conductsivities, Eq. (9.17) can be generalized:

$$k_i = \sum_{i=1}^{n} k_i \phi_i = \sum_{i=1}^{n} k_i \phi_i$$

(9.18)

Here $n$ is the number of particle classes within which all particles have about the same shape and conductivities. As shown by de Vries (1973), the thermal conductivity of soils of widely differing compositions can be estimated by Eq. (9.18) with a fair degree of accuracy. The deviations between measured and estimated values of thermal conductivity were reported to be less than 10%, for $k_i/k_o$ ratios smaller than 10 (i.e., for saturated soils) and about 25% for $k_i/k_o$ ratios of the order of 100 (i.e., dry soils). In moist but unsaturated soils, water can be considered as a continuous medium in which soil particles and air are dispersed. Since the ratio $k_i/k_o$ is greater than unity for mineral particles and less than unity for organic matter and air, the errors caused by these different constituents may compensate each other, at least partially. Thus, Eq. (9.18) may still provide a fair estimate of the composite thermal conductivity even of a three-phase soil. The de Vries model can also be applied in two steps to calculate the thermal conductivity of a soil with aggregated particles. In the first step, the conductivity of the aggregates is calculated. In the second step that of the soil as a whole. There are insufficient data to verify this model, however.

The following form of Eq. (9.18) for an unsaturated soil was used by van Beave and Hillel (1975, 1976):

$$k_o = f_a k_w + \frac{\phi}{1 + \phi} k_o + \frac{\phi}{1 + \phi} k_o$$

wherein $k_w$, $k_o$, and $k_a$ are the specific thermal conductivities of each of the soil constituents (water, air, and an average value for the solids, respectively). The factor $k_b$ represents the ratio between the space average of the temperature gradient in the solid relative to the water phase. The factor $k_a$ depends on the array of grain shapes as well as on mineral composition and organic matter content. The $k_b$ factor represents the corresponding ratio for the thermal gradient in the air and water phases.

The dependence of thermal conductivity and diffusivity on soil wetness is illustrated in Fig. 9.1. The influence of latent heat transfer by water vapor in the air-filled pores is proportional to the temperature gradient in these pores. It can be taken into account (van Beave and Hillel, 1976; Hillel, 1977) by adding to the thermal conductivity of air an apparent conductivity due to evaporation, transport, and condensation of water vapor (the so-called vapor enhancement factor). This value is strongly temperature dependent, and rises rapidly with increasing temperature.
The flux of sensible heat associated with liquid water movement in the soil is generally considered negligible.

Given the complexities involved in any attempt to predict a soil's thermal conductivity by calculation based on theory, one might be justified in asking, "Why bother? Why not simply take measurements?" Indeed, one should never depend on theory alone. Measurements are necessary, if only to validate or invalidate theory. However, the task of measuring thermal conductivity presents its own difficulties and complexities. Because of the fact that soil water potential depends on temperature, the development of a temperature gradient generally induces the movement of water as well as of heat. Hence techniques for measuring heat transfer through a soil sample based on steady-state heat flow between two planes maintained at a constant temperature differential involve the risk of changing the sample's internal moisture distribution, and therefore its thermal properties. In the course of measurement, the soil near the warmer plane becomes drier, while that near the cooler plane becomes wetter. Early attempts to measure thermal conductivity (e.g., Smith, 1932) failed to recognize this pitfall as they purported to maintain constant soil moisture conditions during prolonged steady-state heat flow. Hence their results can only be considered approximations at best. While steady-state methods may be sufficiently accurate for

![Diagram](image-url)
measuring thermal conductivity of dry soils, short-term transient heat-flow techniques are preferable in principle, for moist soils.

The principal advantages of using transient-state methods in the measurement of thermal conductivity are that water movement in the soil volume of interest is obviated, or at least minimized, and that the long wait required for attainment of steady-state heat flow is avoided. In addition, whereas steady-state methods are confined practically entirely to the laboratory, transient-state methods are applicable both in the laboratory and in the field. At present one of the most practical of these methods for measuring thermal conductivity in situ is the cylindrical-probe heat source, which can be inserted into the soil to any depth in the field, and can be used in the laboratory as well (de Vries and Peck, 1958; Woodside, 1958). Its use is based on solution of the equation for heat conduction in the radial direction from a line source (Carslaw and Jaeger, 1959):

\[
\frac{\partial T}{\partial t} = \frac{k}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{c} \frac{\partial T}{\partial t}
\]

(9.19)

where \( T \) is temperature, \( t \) time, \( r \) radial distance from the line source of heat, and \( k \) as before, thermal conductivity.

In practice, a cylindrical probe containing a heating wire is embedded in the soil, an electrical current is then supplied, and the rate of temperature rise is measured with a thermocouple or thermistor placed next to the wire. For a short distance from the line source, the rise in temperature \( T - T_0 \) is given by

\[
T - T_0 = \left( \frac{q_t}{4k\pi} \right) \left( c + \ln \frac{r}{r_0} \right)
\]

(9.20)

wherein \( T \) is the measured temperature, \( T_0 \) the initial temperature, \( q_t \) the heat generated per unit time and unit length of heating wire, \( k \) the conductivity, \( c \) a constant, and \( r \) the time. A plot of temperature versus the logarithm of time permits a calculation of \( c \). A correction factor may be necessary to account for the dimensions of the probe (Tuckson and Taylor, 1965).

The thermal diffusivity \( D_k \), instead of the conductivity \( k \), is sometimes desired. It can be defined as the change in temperature produced in a unit volume by the quantity of heat flowing through the volume in unit time under a unit temperature gradient. An alternative definition, easier to perceive, is that the thermal diffusivity is the ratio of the conductivity to the product of the specific heat and density:

\[
D_k = \frac{k}{c_p \rho} = \frac{k}{C_v}
\]

(9.21)

where \( C_v \) is the volumetric heat capacity. As shown in the preceding section, the specific heat and density of both solids and water must be considered when calculating the volumetric heat capacity:

\[
C_v = \rho c_v (1 + \epsilon \varphi)
\]

(9.22)
where \( \rho \) is the density of dry soil, \( c_s \) the specific heat of dry soil, \( c_w \) the specific heat of water, and \( w \) the ratio of the mass of water to the mass of dry soil.

The thermal diffusivity can thus be calculated from prior measurements of thermal conductivity and volumetric heat capacity, or it can be measured directly as described by Jackson and Taylor (1965).

F. Thermal Regime of Soil Profiles

In nature, soil temperature varies continuously in response to the ever-changing meteorological regime acting upon the soil–atmosphere interface. That regime is characterized by a regular periodic succession of days and nights, and of summers and winters. Yet the regular diurnal and annual cycles are perturbed by such irregular episodic phenomena as cloudiness, cold waves, warm waves, rainstorms or snowstorms, and periods of drought. Add to these external influences the soil’s own changing properties (i.e., temporal changes in reflectivity, heat capacity, and thermal conductivity), as the soil alternately wets and dries, and the variation of all these properties with depth, as well as the influences of geographic location and vegetation, and you can expect the thermal regime of soil profiles to be complex indeed. Yet not altogether unpredictable.

The simplest mathematical representation of nature’s fluctuating thermal regime is to assume that at all depths in the soil the temperature oscillates as a pure harmonic (sinusoidal) function of time around an average value. Since nature’s actual variations are not so orderly, this may be a rather crude approximation. Nevertheless, it is an instructive exercise in itself, and when used in conjunction with field data it can lead to a better understanding, and perhaps even provide a basis for the prediction, of a soil’s thermal regime.

Now let us assume that although soil temperature varies differently at different depths in the soil, the average temperature is the same for all depths. We then choose a starting time \( t = 0 \) such that the surface is at the average temperature. The temperature at the surface can then be expressed as a function of time (Fig. 9.2):

\[
T(0,t) = T + A_0 \sin \omega t
\]

(9.23)

where \( T(0,t) \) is the temperature at \( z = 0 \) (the soil surface) as a function of time \( t \), \( T \) is the average temperature of the surface (as well as of the profile), and \( A_0 \) is the amplitude of the surface temperature fluctuation (the range from maximum to minimum to average temperature). Finally, \( \omega \) is the radial frequency, which is \( 2\pi \) times the actual frequency. In the case of
diurnal variation the period is 86,400 sec (24 hr) so \( w = 2\pi / 86,400 = 7.27 \times 10^{-5} \) sec. Note that the argument of the sine function is expressed in radians rather than in degrees.

The last equation is the boundary condition for \( z = 0 \). For the sake of convenience, let us assume that at infinite depth \( (z = \infty) \) the temperature is constant and equal to \( T \). Under these circumstances, temperature, at any depth \( z \) and time \( t \), is also a sine function of time, as shown in Fig. 9.3 (Lettau, 1962; van Wijk, 1963):

\[
T(z,t) = T + A_1 \sin(wt + \phi(z))
\]

(9.24)

in which \( A_1 \) is the amplitude at depth \( z \). Both \( A_1 \) and \( \phi(z) \) are functions of \( z \) but not of \( t \). They can be determined by substituting the solution of Eq. (9.23) in the differential equation \( \partial T / \partial t = D_1 \partial^2 T / \partial z^2 \). This leads to the solution

\[
T(z,t) = T + A_1 (\sin(wt - z/d)) e^{-d}
\]

(9.25)

The constant \( d \) is a characteristic depth, called the damping depth, at which the temperature amplitude decreases to the fraction \( 1/e \) (1/2.718 = 0.37) of the amplitude at the soil surface \( A_0 \). The damping depth is related to the thermal properties of the soil and the frequency of the temperature fluctuation as follows:

\[
d = (2\pi/cn)^{1/2} = (2D_1/cn)^{1/2}
\]

(9.26)

It is seen that at any depth \( z \) the amplitude of the temperature fluctuation \( A_1 \) is smaller than \( A_0 \) by a factor \( e^{-d} \) and that there is a phase shift (i.e., a time shift) of the temperature fluctuation as shown in Fig. 9.3.

**Fig. 9.3.** Idealized daily fluctuation of surface soil temperature, according to the equation:

\[ T = T + A_1 \sin(wt + \phi), \]

where \( T \) is the temperature, \( T \) average temperature, \( A_1 \) amplitude, \( w \) time, and \( \phi \) period of the oscillation (in this case, \( \pi \) refers to the diurnal period of 24 hr).
Fig. 9.3. Idealized variation of soil temperature with time for various depths. Note that at each succeeding depth the peak temperature is damped and shifted progressively in time. Thus, the peak at a depth of 40 cm lags about 12 hr behind the temperature peak at the surface and is only about 1/3 of the latter. In this hypothetical case, a uniform soil was assumed, with a thermal conductivity of $4 \times 10^{-1} \text{ cal/cm sec deg}$ and a volumetric heat capacity of 0.5 cal/cm$^3$ deg.

delay of the temperature peak $= -\pi/d$. The decrease of amplitude and increase of phase lag with depth are typical phenomena in the propagation of a periodic temperature wave in the soil.

The physical explanation for the damping and regard of the temperature waves with depth lies in the fact that a certain amount of heat is absorbed or released along the path of propagation when the temperature of the conducting soil increases or decreases, respectively. The damping depth is related inversely to the frequency, as can be seen from Eq. (9.26). Hence it attains directly on the period of the temperature fluctuation considered. The damping depth is $\sqrt{\frac{\pi}{2}} \times 19$ times larger for the annual variation than for the diurnal variation in the same soil. For example, van Wijk and de Vries (1963) calculated the damping depth for a soil with $k = 2.3 \times 10^{-3}$ cal/cm sec deg and obtained $d = 12$ cm for the diurnal temperature fluctuation and $d = 229$ cm for the annual fluctuation. Whereas at depth $z = d$ the amplitude
is 0.37 as great as the amplitude at the surface, it is only about 0.05 of the surface amplitude at $z = 1d (1 = 36 \text{ cm for the diurnal variation})$ in the case of the soil used by these authors. When an arbitrary zero point $t_0$ is introduced into the time scale, Eq. (9.25) becomes

$$T(t) = T_0 + A_1(\sin(\omega t + \varphi_0 - z/d_1))e^{-\mu d_1}$$

(9.27)

The constant $\varphi_0 = -\omega t_0$ is called the phase constant.

The annual variation of soil temperature down to considerable depth causes deviations from the simplistic assumption that the daily average temperature is the same for all depths in the profile. The combined effect of the annual and diurnal variation of soil temperature can be expressed by

$$\tilde{T}(z,c) = \tilde{T}_0 + A_1(\sin(\omega t + \varphi_1 - z/d_1))e^{-\mu d_1} + A_2(\sin(\omega t + \varphi_2 - z/d_2))e^{-\mu d_2}$$

(9.28)

where the subscripts indicate $y$ and $d$ refer to the yearly and daily temperature waves, respectively. $\tilde{T}_0$ is the annual mean temperature. The daily cycles are now seen to be short term perturbations superimposed upon the annual cycle. Vagaries of weather (e.g., spells of cloudiness or rain) can cause considerable deviations from simple harmonic fluctuations, particularly for the daily cycles. Longer term climatic irregularities can also affect the annual cycle, of course. Also, since the annual temperature wave penetrates much more deeply, the assumptions of soil homogeneity in depth and of the time constancy of soil thermal properties are clearly unrealistic.

**Fig. 9.4.** The soil temperature profile as it might vary from season to season in a front-line region.
An alternative theoretical approach is now possible, with fewer constraining assumptions. It is based on numerical, rather than analytical, methods for solving the differential equations of heat conduction. With the brute force of increasingly powerful digital computers, it is possible to construct and solve mathematical simulation models which allow soil thermal properties to vary in time and space (e.g., in response to periodic changes in soil wetness), so as to account for alternative surface saturation and desiccation and for profile layering, and which also allow boundary conditions (e.g., climatic inputs) to follow more realistic and irregular patterns. The surface amplitude of temperature need no longer be assumed to be an independent variable, but one which itself depends on the energy balance of the surface and thus is affected by both soil properties and above-

Fig. 9.5. Typical variation of temperature with depth at different times of day in summer. (From Sellers, 1965, based on data given by Carlson, 1961.)
soil conditions. Examples of the numerical approach can be found in the published works of Wierenga and deWitt (1973), van Bavel and Hillel (1975, 1976), and Hillel (1977).

Other recent innovations of practical importance involve the development of techniques for monitoring the soil thermal regime more accurately and precisely than previously available techniques. One such innovation is the infrared radiation thermometer for the scanning or remote sensing of surface temperature for both frozen and vegetated soils without disturbance of the measured surface. Knowledge of the surface temperature and its variation in time is important in assessing energy exchange between soil and atmosphere as well as in determining boundary conditions for soil heat transfer.

Another important technique is the use of heat flux plates. These are flat and narrow plates or disks of constant thermal conductivity which allow precise measurement of the temperature difference between their two sides so as to yield the heat flux through them. When embedded horizontally in the soil at regular depth intervals, a series of such heat flux plates can provide a continuous record of heat transfer throughout the profile. These are problems, however. The presence of heat flux plates can distort the flow of heat in the surrounding medium if their thermal conductivity is very different from that of the soil. The experimental error can be minimized by constructing plates of maximal thermal conductivity and minimal thickness, and by calibrating them in a medium with a thermal conductivity close to that of the soil in which they are to be placed. Another problem is that such plates do not allow vapor flow, which can be an important component of heat transfer. Studies demonstrating the use of heat flux plates were reported by Fuchs and Tannen (1968) and by Hajati and Fuchsj (1974).

The soil temperature profile as it might vary from season to season in a frost-free region is illustrated in Fig. 5. The diurnal variation of temperature and the directions of heat flow within a soil profile are illustrated in Fig. 9.3.

Sample Problems

1. Assuming steady-state conditions, calculate the one-dimensional thermal flux and total heat transfer through a 20-cm thick layer of the (thermal conductivity is $3.6 \times 10^{-2}$ cal cm sec deg $\times$ 10 deg cm) and a temperature differential of 10 $^\circ$C is maintained across the sample for 1 hr.

Using Eq. (9.5) in discrete form, we can write:

\[ q_h = A \Delta T \Delta x = 3.6 \times 10^{-2} \text{ cal cm sec deg} \times 10 \text{ deg cm} \]

\[ = 1.8 \times 10^{-1} \text{ cal cm}^{-2} \text{ sec} \]

Total heat transfer is

\[ q_H = 1.8 \times 10^{-1} \text{ cal cm}^{-2} \text{ sec} \times 3600 \text{ sec} = 648 \text{ cal cm}^{-2} \]
2. A thermal flux of $10^{-3}$ cal/cm$^2$ sec is maintained into the upper surface of a 10 cm thick sample, the bottom of which is thermally insulated. Calculate the time rate of temperature change and the total temperature rise per hour if the bulk density is 1.2 gm/cm$^3$ and the specific heat capacity is 0.6 cal/gm deg.

For this case of heat flow, we use a discrete form of Eq. (9.16):

$$dT \over dt = (Q_0 \Delta V)(1/\rho_{soil})$$

Using the data provided, the time rate of temperature change:

$$dT \over dt = (10^{-3} \text{ cal/cm}^2 \text{ sec} \times 10 \text{ cm} \times 1.2 \text{ gm/cm}^3 \times 0.6 \text{ cal/gm deg})^{-1} = 1.39 \times 10^{-3}$$

Total temperature rise is $1.39 \times 10^{-3} \text{ deg/ sec} \times 3600 \text{ sec/hr} = 9.5 \text{ C/hr}$.

3. Calculate the volumetric heat capacity $C$ of a soil with a bulk density of 1.46 gm/cm$^3$ when completely dry, and completely saturated. Assume that the density of solids is 2.60 gm/cm$^3$ and that organic matter occupies 10% of the solid matter by volume.

First calculate the volume fraction of pores (the porosity):

$$f = \rho_{soil} - \rho_{m} \rho_{s} = (1.46 - 2.60) \text{ gm/cm}^3 - 2.60 \text{ gm/cm}^3 = 0.44$$

Hence the volume fraction of solids is $1 - 0.44 = 0.56$. Since organic matter constitutes 10% of the soil’s solid phase, the volume fraction of mineral matter is

$$f_{m} = 0.56 \times 0.9 = 0.504$$

The volume fraction of organic matter is

$$f_{o} = 0.56 \times 0.1 = 0.056$$

The volumetric heat capacity can be calculated using Eq. (9.15):

$$C = f_{m}C_{m} + f_{w}C_{w} + f_{o}C_{o}$$

where $f_{m}$, $f_{o}$, and $f_{w}$ are the volume fractions of mineral matter, organic matter, and water, respectively; and $C_{m}$, $C_{o}$, and $C_{w}$ refer to heat capacities of the same constituents (generally 0.48 cal/cm$^3$ deg for mineral matter, 9.6 cal/cm$^3$ deg for organic matter, and 1 cal/cm$^3$ deg for water). Accordingly, when the soil is completely dry,

$$C = (0.48 \times 0.504) + (0.60 \times 0.056) = 0.24 + 0.03 = 0.27 \text{ cal/cm}^3 \text{ deg}$$

When the soil is saturated, its volumetric fraction of water equals the porosity. Thus

$$C = 0.27 \text{ cal/cm}^3 \text{ deg} + 0.44 \times 1 \text{ cal/cm}^3 \text{ deg} = 0.71$$

Note: We have completely neglected the heat capacity of air, since it is too small to make any significant difference (see Table 9.1).
4. The daily maximum sub-surface temperature is 40°C and the minimum is 10°C. Assuming that the diurnal temperature wave is symmetrical, that the mean temperature is equal throughout the profile (with the surface temperature equal to the mean value at 6 a.m. and 6 p.m.), and that the "damping depth" is 9 cm, calculate the temperature at noon and midnight for depths 0, 5, 10, and 20 cm.

Since the temperature range is 30°C and the mean (T) is 25° C, the amplitude at the surface (A₀), the maximum value above the mean, is 15.

We use Eq. (9.25) to calculate the temperature T at any depth z and time t:

\[ T(z,t) = T₀ + A₀ \sin(\omega t - \pi/2)/\epsilon^z \]

where \( \epsilon \) is the radial frequency (2π/24 hr) and \( \omega \) is the "damping depth" at which the temperature amplitude is 1/e of \( A₀ \). Note: The radial angle is expressed in radians rather than as degrees.

At depth zero (the soil surface): Noon-time temperature (6 hr after \( T₀ = T₀)\):

\[ T(0.6) = 25 + 15 \times |\sin(\pi/2 - 0)|/\epsilon^0 = 25 + 15 = 40°C \]

Midnight temperature (18 hr after \( T₀ = T₀)\):

\[ T(0.18) = 25 + 15 \times |\sin(3\pi/2 - 0)|/\epsilon^0 = 25 - 15 = 10°C \]

At depth 7 cm: Noon-time temperature:

\[ T(5.9) = 25 + 15 \times |\sin(\pi/2 - 5/10)|/\epsilon^7.1 \approx 15 \]

Midnight temperature:

\[ T(5.98) = 25 + 15 \times |\sin(\pi/2 - 5/10)|/\epsilon^7.1 \approx 10°C \]

At depth 10 cm (for damping depth): Noon-time temperature:

\[ T(9.6) = 25 + 15 \times |\sin(\pi/2 - 1)|/\epsilon = 25 + 15 \times |\sin(0.57)|/\epsilon = 25 + 15 \times 0.3296/2.718 \approx 27.98°C \]

Midnight temperature:

\[ T(10.18) = 25 + 15 \times |\sin(3\pi/2 - 1)|/\epsilon = 25 + 15 \times 0.3296/2.718 \approx 22.92°C \]
At depth 20 cm: Noon temperature:
\[ T(20.6) = 25 + 15 \times \sin(1.57 - 20/10) \times e^{20.10} \]
\[ = 25 + 15 \times [\sin(-0.47)]/e^2 \]
\[ = 25 + 15 \times (-0.41687/7.39) = 25 - 0.85 = 24.15 \, ^\circ C \]

Midnight temperature:
\[ T(20.18) = 25 + 15 \times [\sin(4.71 - 2)]/7.39 \]
\[ = 25 + 15 \times (0.41687/7.39) = 25.85 \, ^\circ C \]

Note: At a depth of 20 cm the phase shift is so pronounced that the temperature at midnight is actually higher than at noon time.

A useful exercise for students at this point is to calculate and to plot the sinusoidal variation of temperature at each depth so as to observe how the phase shift (time lag of maximum and minimum values) increases and the amplitude decreases with depth.